

Linear interpolated multi-objective economic emission scheduling using grey wolf optimizer: a strategic balance and solution with diverse load pattern

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Received 8 February 2021
Revised 11 September 2021
Accepted 1 November 2021

Abstract

Purpose – The purpose of this paper is to focus on the cost-effective and environmentally sustainable operation of thermal power systems to allocate optimum active power generation resultant for a feasible solution in diverse load patterns using the grey wolf optimization (GWO) algorithm.

Design/methodology/approach – The economic dispatch problem is formulated as a bi-objective optimization subjected to several operational and practical constraints. A normalized price penalty factor approach is used to convert these objectives into a single one. The GWO algorithm is adopted as an optimization tool in which the exploration and exploitation process in search space is carried through encircling, hunting and attacking.

Findings – A linear interpolated price penalty model is developed based on simple analytical geometry equations that perfectly blend two non-commensurable objectives. The desired GWO algorithm reports a new optimum thermal generation schedule for a feasible solution for different operational strategies. These are better than the earlier reports regarding solution quality.

Practical implications – The proposed method seems to be a promising optimization tool for the utilities, thereby modifying their operating strategies to generate electricity at minimum energy cost and pollution levels. Thus, a strategic balance is derived among economic development, energy cost and environmental sustainability.



This work is supported by the Department of EEE, Shri Vishnu Engineering College for Women (Autonomous), Bhimavaram, Andhra Pradesh, India.

Funding: This research received no funding from any agency.

Originality/value – A single optimization tool is used in both quadratic and non-convex cost characteristics thermal modal. The GWO algorithm has discovered the best, cost-effective and environmentally sustainable generation dispatch.

Keywords Optimal control, Numerical analysis, Multiobjective optimization, Design optimization methodology, Diverse load pattern, Economic emission sustainable dispatch (EESD), Grey wolf optimizer, Linear interpolated price penalty model, Metaheuristic optimization

Paper type Research paper

Nomenclature

N_s	= number of thermal units;
T	= total number of the scheduling period in “hour”;
t_k	= subinterval duration in “hour”;
F	= total generation cost (\$/h);
E	= total amount of ER (lb/h);
a_i (\$/h), b_i (\$/MWh), c_i (\$/MW ² h)	= coefficients of the cost curve of i th thermal unit;
d_i (\$/h), g_i (rad/MW)	= valve point effect coefficient of i th thermal unit;
α_i (lb/h), β_i (lb/MWh), γ_i (lb/MW ² h), η_i (lb/l), δ_i (1/MW)	= emission curve coefficients of i th thermal plant;
P_{si}	= generation of the i th thermal unit in MW;
P_{si}^{\min} , P_{si}^{\max}	= minimum and maximum generation limit of i th thermal unit in MW;
P_D	= total power demand in MW;
P_L	= total network loss in MW;
B_{mn} , B_{m0} , B_{00}	= loss coefficients;
$P_{m, k}$, $P_{n, k}$	= power generation of the m th and n th index of plants in MW;
h_k	= linear interpolated price penalty factor at k th interval \$/lb;
P_{sj}^L	= P_{sj}^L lower and upper bound of the j th prohibited operating zone in MW;
ND	= number of the prohibited discharge zone;
UR_i , DR_i	= upper and down ramp limit of the i th thermal plant in MW/h;
\vec{D}	= position vector of each hunter from any other hunters;
\vec{X}_{prey}	= position vector of the prey;
\vec{X}_{wolf}	= position vector of the grey wolf;
t	= current iteration;
\vec{A} , \vec{C}	= coefficient vectors;
\vec{a}	= acceleration vector;
\vec{r}_1 , \vec{r}_2	= random vectors in [0, 1]; and
$iter\ max$	= maximum iteration.

1. Introduction

1.1 Background of study

The power sector is pivotal in fueling all other sectors to function and grow through a constant and reliable electricity supply. At the same time, rapid urbanization and

industrialization demand electricity additionally. In a developing country like India, the increasing demand is governed by coal-based thermal plants. Hence, logically, to reduce the gap between the rising demand and limited supply of power, the country must establish new and more power plants or improve the existing one's operation. The optimum system operational setting concerns the economy of operating, system security and utilization of energy resources. Numerous research studies have analyzed thermal plants' optimal operation considering either single or multiple objectives under static and dynamic environments. The single objective optimization aims to minimize the total operating cost of active power generation to meet the demand while satisfying various constraints and transmission loss. A large turbine has multiple steam valves; therefore, the valve point effect is included in the fuel cost (FC), contributing to non-convexity in the cost function. In contrast, the multiobjective operational problem intends to optimize both the FC and emission release (ER) simultaneously. Various classical, metaheuristic and hybrid optimization algorithms that have ascertained solutions for economic and combined economic emission operation problems are briefly discussed in this context.

1.2 Role of metaheuristic and hybrid algorithms in solving cost-effective dispatch

Usually, a cost-effective dispatch (CED) problem has been solved by using many classical methods. A Hopfield neural network approach is used in this category to minimize total FC (Naser *et al.*, 2005). The enhanced augmented Lagrange Hopfield network has been exercised to find optimal solutions corresponding to the chosen fuel types (Dieu and Ongsakul, 2008). A backtracking search algorithm is proposed to find the most elegant solution within a short computation period. However, the valve point loading effect and prohibited operating zones have been included (Modiri-Delshad *et al.*, 2016). An improved chicken swarm optimization has been adopted for saving many FCs. The non-linearity of power generation units' cost characteristics and inequality constraints have suitably been handled (Li *et al.*, 2018). Spea (2020) has applied a newly developed crow search algorithm (CSA) to perceive excellent convergence characteristics, and results confirm the robustness and effectiveness of solving practical economic load dispatch (ELD) problems.

In the optimization environment hybridizing, two algorithms have one global, and another local search property is getting familiar. A particle swarm optimization (PSO) has integrated with local random search (Selvakumar and Thanushkodi, 2007), a genetic algorithm, pattern search and sequential quadratic programming (SQP) techniques are unified (Alsumait *et al.*, 2010), differential evolution (DE) with PSO (Parouha and Das, 2016) and DE through sine cosine algorithm (Babar *et al.*, 2020). Thus, the exploration capability has been enhanced and achieved significant improvement in convergence time.

1.3 Environmental impact and solution for multiobjective optimization

Environmental sustainability needs to optimize both the generation cost and the cost of controlling emissions from the thermal unit operation for ecological sustainability. Some of the research works that have emphasized the gist of emission control in thermal power plants by the optimum allocation of the generation are summarized as follows: a radial basis function neural network (Kulkarni *et al.*, 2002), DE (Mandal and Chakraborty, 2018) and PSO (Pao-La-Or *et al.*, 2010). A multiobjective DE (MODE) algorithm is exercised to solve economic, environmental dispatch in which a Pareto-based approach is introduced to implement the selection of the best individuals (Basu, 2011). Combined economic and emission dispatch (CEED) aims to optimize the bi-objectives simultaneously using the gravitational search algorithm (GSA) for optimum generation scheduling (Güvenç *et al.*, 2012). Rizk-Allah *et al.* (2018) presented a parallel hurricane optimization algorithm (PHOA)

to quickly optimize the economic and emission objectives to reach Pareto optimal solutions. [Rezaie et al. \(2018\)](#) used a chaotic improved harmony search algorithm (CIHSA) to solve the CEED.

Matthew and Nicodemus have hybridized the artificial bee colony algorithm (ABC) with PSO to optimize CEED simultaneously. [Zhang et al. \(2013\)](#) have formulated an enhanced multiobjective cultural algorithm (EMOCA) by combining cultural algorithm and PSO to compromise FC and ER. [Liang et al. \(2018\)](#) have exercised the bat algorithm, and [Bhargava and Yadav \(2020\)](#) have blended the DE-CSA to obtain a trade-off between FC and ER.

1.4 Dynamic combined economic emission dispatch and its solution

Non-dominated sorting genetic algorithm-II (NSGA-II) ([Basu, 2008](#)), improved bacterial foraging algorithm (IBFA) ([Pandit et al., 2012](#)), MODE algorithm ([Jiang et al., 2013](#)), hybrid DE and DE-SQP and hybrid PSO-SQP ([Elaiw et al., 2013](#)), chemical reaction optimization (CRO) algorithm ([Roy and Bhui, 2015](#)), new modified non-dominated sorting genetic algorithm-II (MNSGA-II) ([Zhu et al., 2016](#)) and new enhanced harmony search (NEHS) algorithm ([Li et al., 2019](#)) were attempted to improve the computational efficiency and offer the best compromise solution for dynamic EESD.

1.5 Role of grey wolf optimizer as emerging optimization tool

Grey wolf optimizer (GWO) is a newly emerged optimization algorithm developed by [Mirjalili et al., 2014](#) and detailed in Section 3. The GWO algorithm is tested with standard functions and indicates superior exploration and exploitation characteristics than other swarm intelligence techniques. Further, the GWO has successfully applied for solving various engineering optimization problems such as parameter estimation in surface waves ([Song et al., 2015](#)), tuning approach for fuzzy control-based servo systems with reduced parametric sensitivity ([Precup et al., 2017](#)), economic load dispatch problems ([Pradhan et al., 2016](#)) and dynamic economic load dispatch problem ([Sattar et al., 2019](#)).

1.6 Research gap and challenge

There are several approaches to blend the conflicting objectives in the multiobjective optimization environment. Particularly in the thermal power system's (TPS) combined economic emission operation, the weighted aggregation is used widely. The weight factor of an objective is chosen in proportion to the relative importance of the aim. It is observed that the weighted sum approach has given equal preference for both the FC and ER and optimized upon the satisfaction of equality constraints; apart from that, the trade-off procedure is not directly connected with load demand. The modified price penalty factor approach is exercised consistently to obtain the compromised solution in CEED.

In contrast with the weighted aggregation method, the modified price penalty factor approach is directly related to the load demand, but maximum capacity is often greater than demand. Hence, this procedure yields the approximate price penalty factor, and the solution might be hypothetically in global optima. A set of non-dominated solutions is obtained for CEED popularly using the multiobjective evolutionary algorithms (MOEA) with sorting techniques and determining the Pareto optimal fronts. It is observed that the MOEA has obtained non-dominated solutions, which are not directly related to load demand. The relation between FC and ER release of thermal plant operation is highly conflicting, and balancing the same is a real challenge.

1.7 Highlights and organization of the paper

In this paper, an accurate linear interpolated price penalty model is developed based on simple analytical geometry, an equation that blends two non-commensurable objectives perfectly. A solution repair strategy is adopted to satisfy power balance constraints. Moreover, most of the swarm intelligent techniques used to solve optimization problems cannot control the leader over the entire period. Therefore, the emerging GWO is aimed to apply for solving economic environmentally sustainable dispatch. The GWO algorithm has the following versatile properties inherently:

- self-organization;
- natural leadership hierarchy;
- support in decision-making;
- fitness of the potential solution is the average of the first three best solution; and
- few control parameters.

Additionally, GWO's control parameters are turned to search with the more global solution and linearly reach a steady and perfect local value.

This paper is organized into six sections. Section 2 describes all thermal scheduling problems, whereas Section 3 briefs the GWO algorithm. Section 4 deals with application of a GWO algorithm for finding an optimal generation schedule. The numerical simulation results of various case studies have been presented and compared in Section 5. Finally, the conclusion and possible future work are summarized and suggested in Section 6.

2. Problem formulation

The optimum generation schedule of the utility is a complex combinatorial optimization problem that aims to supply a demand for affordable costs. Generally, linear, quadratic and cubic cost functions (CCFs) have been used in the revenue analysis. Though the CCF is legitimate, cost analysts crave the quadratic cost function (QCF) because the solution procedure of CCF starts with a hypothetical initial value. In contrast, the QCF involves a straightforward approach (Vali, 2014). Therefore, the optimum operation of the thermal entity has been established with QCF realistically.

2.1 State-of-the-art dispatch models

In the power sector, the thermal power plant takes an overwhelming portion of the entire generation. It causes gaseous emission and air pollution. Hence, the optimum generation schedule considers two objective functions: the total FC and the ER. Therefore, combined economic emission dispatch (ED) must carry to attain the trade-off between FC and ER. For this purpose, various dispatch models are developed based on input–output characteristics using a simple recursive procedure (Mandal and Chakraborty, 2018; Basu, 2008).

2.1.1 Cost-effective dispatch. This dispatch has intended to minimize FC alone. Hence, the cost function optimized over a scheduling period under a dynamic load environment is mathematically defined in (1). In which, the scheduling period is restricted into one for static load pattern as follows:

$$\text{Minimize } F = \sum_{k=1}^T \sum_{i=1}^{N_s} t_k [f_{i,k}(P_{si,k})] \quad (1)$$

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At this point, each thermal plant's FC characteristic is composed of one or more quadratic segments as a function of the active power generation and is represented by (2):

$$f_{i,k}(P_{si,k}) = a_i + b_i P_{si,k} + c_i P_{si,k}^2 \quad (2)$$

Practically, the thermal power plant facilitates multi-valve steam turbines for flexible operation. Then, the valve point loading effect is designated as sinusoidal function and superposition with quadratic cost characteristics. Therefore, the cost characteristic becomes non-convex and is defined by (3):

$$\text{Minimize } F = \sum_{k=1}^T \sum_{i=1}^{N_s} t_k \left[a_i + b_i P_{si,k} + c_i P_{si,k}^2 + \left| d_i \sin \left\{ g_i \left(P_{si}^{\min} - P_{si,k} \right) \right\} \right| \right] \quad (3)$$

2.1.2 Emission dispatch. The reduction of ER associated with thermal power generation is a paramount process from environmental conservation. Therefore, the thermal plant's ER to be minimized over a scheduling period under a dynamic load pattern has mathematically defined in (4). Wherein, the scheduling period is limited to one for the static load as follows;

$$\text{Minimize } E = \sum_{k=1}^T \sum_{i=1}^{N_s} t_k [e_{i,k}(P_{si,k})] \quad (4)$$

At this juncture, the thermal unit's emission supplements the usual cost function and is described as the sum of a quadratic function in (5):

$$e_{i,k}(P_{si,k}) = \alpha_i + \beta_i P_{si,k} + \gamma_i P_{si,k}^2 \quad (5)$$

The realistic operation is mathematically approximated to a QCF combined with an exponential function in (6):

$$\text{Minimize } E = \sum_{k=1}^T \sum_{i=1}^{N_s} t_k \left[\alpha_i + \beta_i P_{si,k} + \gamma_i P_{si,k}^2 + \eta_i \exp(\delta_i P_{si,k}) \right] \quad (6)$$

2.1.3 Combined economic emission dispatch. The optimum generation schedule with absolute minimum FC is not anymore the only criterion. Additionally, environmental considerations have become one of the major management concerns. Therefore, the problem is formulated as a multiobjective optimization and described as follows:

- Static load pattern:

$$\text{Minimize } \sum_{i=1}^{N_s} \{ [f_{i,k}(P_{si,k})], [e_{i,k}(P_{si,k})] \} \quad (7)$$

- Dynamic load pattern:

$$\text{Minimize } \sum_{k=1}^T \sum_{i=1}^{N_s} t_k \{ [f_{i,k}(P_{si,k})], [e_{i,k}(P_{si,k})] \} \quad (8)$$

2.2 System constraints

CEED is intended to minimize the operating cost of the thermal units with reduced ER through an optimal generation schedule. Therefore, the constraints imposed on the optimal operation of the TPS, such as active power balance, generation limits, prohibited operating zone and ramp rate limits, are discussed in this section (Mandal and Chakraborty, 2018; Jiang et al., 2013).

2.2.1 Power balance. It is an equality constraint and states that the algebraic sum of the optimal active power generation of thermal units, load demand and transmission loss at any interval should be equal to zero as follows:

$$\sum_{i=1}^{N_s} P_{s,ik} - P_{D,k} - P_{L,k} = 0; \quad k \in T \quad (9)$$

The following redefined loss formula gives the transmission power loss in terms of the coefficient (B_{mn}):

$$P_{L,k} = \sum_{m=1}^{N_s} \sum_{n=1}^{N_s} P_{m,k} B_{mn} P_{n,k} + \sum_{m=1}^{N_s} B_{m,0} P_{m,k} + B_{00}; \quad k \in T \quad (10)$$

2.2.2 Power generation limits. It is an inequality constraint, and the active power generation of each thermal unit is between its upper and lower operating region as follows:

$$P_{si}^{\min} \leq P_{si} \leq P_{si}^{\max} \quad i = 1, 2, 3 \dots N_S \quad (11)$$

2.2.3 Prohibited operating zone. The thermal units' operations have been restricted for a specific region is formulated as prohibited operating zone (POZ) constraint and is given as follows:

$$\begin{cases} P_{sj}^{\min} \leq P_{sj,k} \leq P_{sj,1}^L \\ P_{sj,m-1}^U \leq P_{sj,k} \leq P_{sj,m}^L; \quad m = 2, 3, \dots, ND_j \\ P_{sj,m}^U \leq P_{sj,k} \leq P_{sj}^{\max}; \quad m = ND_j \end{cases} \quad (12)$$

2.2.4 Ramp rate limit. The sudden rise and fall of the i th thermal unit have been expelled is represented as ramp rate limit (RRL) by (13):

$$\begin{cases} P_{si,k} - P_{si,k-1} \leq UR_i; \quad \text{if generation increases} \\ P_{si,k-1} - P_{si,k} \leq DR_i; \quad \text{if generation decreases} \end{cases} \quad (13)$$

2.3 Proposed dispatch model

In the CEED, the optimization is formulated as a multiobjective problem with conflict and non-commensurable objectives. It transformed into a single objective optimization problem by using an efficient approach, which blends the emission with the typical FC's and is detailed in this section.

2.3.1 Modified price penalty factor. Usually, the bi-objective of CEED is handled using a modified price penalty factor approach detailed as follows. Generally, the price penalty factor has defined as the ratio between the maximum FC and the maximum emission of the corresponding generator. The computation procedure is as follows (Mandal and Chakraborty, 2018; Moorthy *et al.*, 2015; Moorthy *et al.*, 2015):

Step 1. The average full-load FC (f_{av}), i.e. FC per unit of power when the entity is at its total capacity, has been computed as follows:

$$f_{av} = \frac{F(P_{si}^{\max})}{P_{si}^{\max}} \quad (14)$$

Step 2. The average emission (e_{av}) of each thermal unit at its maximum output has been calculated as follows:

$$e_{av} = \frac{E(P_{si}^{\max})}{P_{si}^{\max}} \quad (15)$$

Step 3. The ratio between f_{av} and e_{av} was determined as follows, and it is called h_{\max} :

$$h_{\max} = \frac{f_{av}}{e_{av}} \quad (16)$$

Step 4. According to h_{\max} , the thermal units have ranked in ascending order.

Step 5. Then, each unit's corresponding full-load capacity is added one at a time until it ($\sum P_{s,i}^{\max} \geq P_{D,k}$) has been discerned.

Step 6. In Step 5, the price penalty factor (h_{\max}) corresponding to the last unit is used to trade-off two conflict objectives.

In this procedure, the maximum capacity of thermal units is often higher than demand; it yields an approximate value. Hence, an accurate model is necessary to determine the no-inferior solution, and this drawback has been rectified by incorporating a simple mathematical technique with the usual procedure.

2.3.2 Linear interpolation. Linear interpolation is a mathematical technique to enhance the process of finding a function [$f_1(x)$] that takes on specified values (x) at specified points (D). Figure 1 explains the generalized analytical geometry procedure that has been used to derive a function that takes on specified values at precise locations. The mathematical linear interpolation model is given by (17):

$$f_1(x) = f(x_o) + \left(\frac{f(x_1) - f(x_o)}{x_1 - x_o} \right) * (x - x_o) \quad (17)$$

2.3.3 Linear interpolated economic environmentally sustainable dispatch. Let P_{s1} is the maximum capacity of a unit at that moment by adding the same cause sum—total exceeds the load demand P_{Dk} , and its corresponding price penalty factor is h_1 . The maximum capacity P_{s0} is the predecessor, and the associated price penalty is h_o . Then, the normalized price penalty factor can be determined using (18):

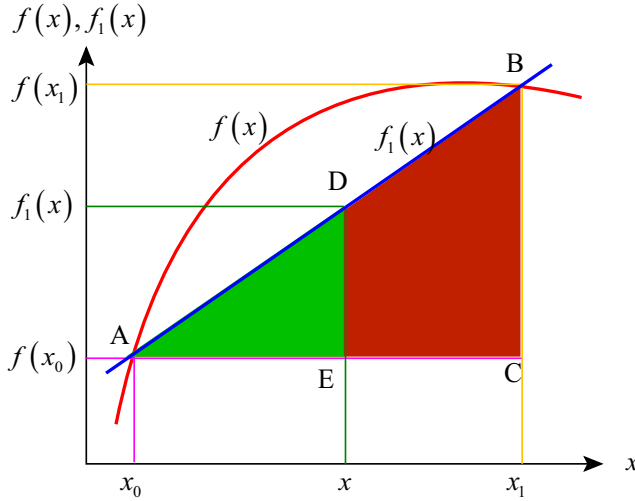


Figure 1.
Linear interpolation

$$h_k = h_o + \left(\frac{h_1 - h_o}{P_{s1} - P_{so}} \right) * (P_{Dk} - P_{so}^{so}) \quad (18)$$

Now, the two objective functions are combined using h_k ; then, the objective function of the EESD problem is defined as follows:

$$\text{Minimize } \{F(P_{si,k}) + h_k * E(P_{si,k})\} \quad (19)$$

3. Grey wolf optimizer modeling

3.1 Mathematical model

Mirjalili (2014) has developed the GWO algorithm inspired by the governance structure and foraging method of grey wolves in nature. Grey wolves are segmented as alpha (α), beta (β), delta (δ) or omega (ω) in their population; the alpha is most dominant, whereas deltas and omegas control the remaining wolves. The three pertinent attitudes of grey wolves are encircling, hunting and assaulting prey, and these are mathematically derived as an optimization algorithm. Generally, the alpha wolf presides over the hunting in association with beta and delta wolves. Therefore, three of the best candidate solutions are designated as alpha, beta and delta wolves during the iteration, whereas the remaining wolves are termed as omega and keep posted consequently. This behavior is mathematically modeled as follows:

$$\vec{X}(t+1) = \frac{\vec{X}_1(t) + \vec{X}_2(t) + \vec{X}_3(t)}{3} \quad (20)$$

The position vector of the grey wolf for the next iteration is given by (21):

$$\left. \begin{aligned} \vec{X}_1 &= \vec{X}_{alpha} - \vec{A}_1 \cdot \vec{D}_{alpha} \\ \vec{X}_2 &= \vec{X}_{beta} - \vec{A}_2 \cdot \vec{D}_{beta} \\ \vec{X}_3 &= \vec{X}_{delta} - \vec{A}_3 \cdot \vec{D}_{delta} \end{aligned} \right\} \quad (21)$$

The position vector of each hunter from any other hunters and is specified by (22):

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$$\left. \begin{aligned} \vec{D}_{alpha} &= |\vec{C}_1 \cdot \vec{X}_{alpha} - \vec{X}| \\ \vec{D}_{beta} &= |\vec{C}_1 \cdot \vec{X}_{beta} - \vec{X}| \\ \vec{D}_{delta} &= |\vec{C}_1 \cdot \vec{X}_{delta} - \vec{X}| \end{aligned} \right\} \quad (22)$$

While indicating wolves' movement to assault the prey, the coefficient vectors "A" in (21) and "C" in (22) play an essential role in mutating the potential neighbors and computing using (23) and (24), respectively (Kadali *et al.*, 2018) as follows:

$$\vec{A} = 2\vec{a} \cdot \vec{r}_1 - \vec{a} \quad (23)$$

$$\vec{C} = 2\vec{r}_2 \quad (24)$$

Significantly, the magnitude of vector "A" decides the convergence tendency of GWO. If it satisfies (25), the hunter toward the prey; otherwise, it diverges from the victim if the measure meets (26), and hopefully, a fitter prey has found:

$$|\vec{A}| \leq 1 \quad (25)$$

$$|\vec{A}| \geq 1 \quad (26)$$

3.2 Strategic balance

To provide an excellent strategic balance between exploration and exploitation, approaching the prey decreases from high value 2 to 0 linearly throughout iterations. Hence, vector "A" fluctuates randomly, ranging from $-2a$ to $2a$, and at the same time, vector "C" contains a random value in $[0, 2]$. It provides weight to the hunter for emphasizing or deemphasizing the position in (22). Therefore, half of the iterations are devoted to exploration in (26), and the rest to exploitation in (25). This mechanism assists GWO to provide exquisite exploration, local minima avoidance and exploitation simultaneously.

4. Implementation of grey wolf optimizer for economic emission sustainable dispatch

The GWO's control parameters setting and the strategy used to evade premature convergence while optimizing the TPS's generation schedule for minimum FC considering the environmental aspect are detailed.

4.1 Initialization of wolves and structure of solutions

Step 1. Read the system data and initialize GWO parameters such as search agents pack size, the maximum allowable iterations (*Iter max*) and the value of the vector (a , A and C) using (23) and (24).

Step 2: The thermal unit's active power generation is a decision variable, and it represents the wolves' position to be evolved. These wolves have been generated randomly based on the pack's size using the mathematical expression (27) as follows:

$$P_{g,i} = rand^* \left(P_{gi}^{\max} - P_{gi}^{\min} \right) + P_{gi}^{\min} \quad (27)$$

Step 3: Thus, the wolves' position is represented as an array for a solution of the EESD, and then, the initial position of the candidate solution (X^0) is structured as follows:

$$X^0 = \left[P_{g1}^1 \dots P_{g1}^{SP} \ P_{g2}^1 \dots P_{g2}^{SP} \ \dots \ P_{gi}^1 \dots P_{gi}^{SP} \ \dots \ P_{gN}^1 \dots P_{gN}^{SP} \right] \quad (28)$$

4.2 Evaluation of fitness

Step 1. The objective function is computed using (29) for the wolves' initial location:

$$\text{Minimize} \left\{ f\left(P_{g1}^1\right) \dots f\left(P_{g1}^{SP}\right) f\left(P_{g2}^1\right) \dots f\left(P_{g2}^{SP}\right) \dots f\left(P_{gi}^1\right) \dots f\left(P_{gi}^{SP}\right) f\left(P_{gN}^1\right) \dots f\left(P_{gN}^{SP}\right) \right\} \quad (29)$$

Step 2. An augmented objective function (AOF) is derived using (30) to handle the equality constraint violation. The calculated objective value is inflated with the absolute value in breach of equality constraint violation by multiplying a high valued scalar multiplier as follows:

$$AOF = \left(\text{objective} + 1000^* \left| \sum_{i=1}^N P_{gi} - (P_D + P_L) \right| \right) \quad (30)$$

Step 3. The fitness value of all individuals of the current candidate solution matrix (X^0) is calculated using (31):

$$\text{fitness}_i = \{AOF\} \quad (31)$$

4.3 Modifying agent position for the optimal solution

Step 1. The fitness of i th individuals has better knowledge about the potential location of prey. Therefore, the first three best (minimum) solutions have been categorized as alpha, beta and delta wolves, respectively, and oblige the other wolves (omega) in the pack. It is represented using (32) for further process as follows:

$$\begin{cases} P_g^{\text{alpha}} = f^1(P_g) = \text{First Minimum} \\ P_g^{\text{beta}} = f^2(P_g) = \text{Second Minimum} \\ P_g^{\text{delta}} = f^3(P_g) = \text{Third Minimum} \end{cases} \quad (32)$$

Step 2. The position of the i th agent should be updated using (20). Each search agent's location represents a potential solution comprising an active power generation of the EESD. Each agent's new site may violate allowable ranges, and it is limited to the respective boundary.

4.4 Fitness re-estimation and termination criterion

Step 1. With the new position of each control variable, the AOF is calculated using (24), and the global solution is identified using (25).

Step 2. Update the wolves' values and vectors (a , A and C).

Step 3. If $Iter < Iter\ max$, the position of the wolves is modified using (26). Otherwise, the GWO terminates.

4.5 Constraints handling

The efficient constraints handling strategy detailed here is the successful critical application of the proposed GWO algorithm for dealing with the EESD problem.

4.5.1 *Power balance constraint.* It was handled effectively without depriving the computational effort of the GWO algorithm, and the step-by-step procedure is described as follows:

Step 1. N_d was chosen as a dependent thermal unit.

Step 2. Its generation at the k th interval (i.e. $P_{gd,k}$) was computed using (33):

$$\begin{aligned}
 B_{dd}P_{gd,k}^2 + \left(2 \sum_{m=1}^{(N-1)} B_{d,m}P_{m,k} - 1 \right) P_{gd,k} \\
 + \left(\sum_{m=1}^{(N-1)} \sum_n^{(N-1)} P_{m,k}B_{mn}P_{n,k} + \sum_{m=1}^{(N-1)} B_{m,0}P_{m,k} \right. \\
 \left. - \sum_{\substack{m=1 \\ m \neq d}}^{(N-1)} P_{m,k} + B_{00} + P_{D,k} \right) = 0; \quad k \in T
 \end{aligned} \tag{33}$$

Step 3. The positive root was considered a generation of a dependent thermal unit (N_d).

Step 4. If the dependent generation is not satisfied, (11) at the initial stage itself, repeat installation of $(N-1)$ units as far as it meets the operating region.

Step 5: If the dependent generation is unsatisfied, (11) the mutated set of peers is abandoned afterwards. The procedure was applied in its earlier solution as far as it meets the operating region.

4.5.2 *Inequality constraint.* If the newly generated decision variable violates, it is an available operating range and is handled appropriately. If any power generation is less than the minimum level, it is fixed to the minimum value. Similarly, if it is higher than the maximum level, it is assigned its upper value as follows:

$$P_{gi,k} = \begin{cases} P_{g,i}^{\min} & \text{if } P_{gi,k} < P_{g,i}^{\min} \\ P_{g,i} & \\ P_{g,i}^{\max} & \text{if } P_{gi,k} > P_{g,i}^{\max} \end{cases} \quad i \in N_g; \quad k \in T \tag{34}$$

4.5.3 *Prohibited operating zones.* Considering POZs of thermal plants, the generation limit has been managed using (35) as follows:

$$P_{g,i} = \begin{cases} P_{g,m}^L & \text{rand} \leq 0.5 \\ P_{g,m}^U & \text{rand} > 0.5 \end{cases} \quad m = 2, 3, \dots, ND_i \quad (35)$$

Grey wolf optimizer

4.5.4 *Ramp rate limit.* The operating limits of thermal units with RRL can be handled as follows:

$$\left. \begin{aligned} P_{gi,k}^{\max} &= \min \left\{ P_{gi}^{\max}, (P_{gi,k-1} + UR_i) \right\} \\ P_{gi,k}^{\min} &= \max \left\{ P_{gi}^{\min}, (P_{gi,k-1} - DR_i) \right\} \\ P_{gi,k}^{\min} &\leq P_{gi,k} \leq P_{gi,k}^{\max} \end{aligned} \right\} \quad (36)$$

4.6 Computational time complexity of grey wolf optimizer

The time complexity (TC) of the GWO to take the EESD problem is computed based on the pseudo code and implementation procedure. It comprises four phases, and each computational phase time is expressed as follows (Moorthy *et al.*, 2015):

- (1) *Initialization phase (IP):* There are two nested loops; one iterates N time (wolves' size), and the other iterates with d (dimensionality of the control variable) as follows:

$$TC(IP) = O(N*d)$$

- (2) *GWO algorithm phase (GAP):* It comprises GWO operator, i.e. encircling and attacking prey and social hierarchy. It has a *While* loop with a single *For* loop in it. The *While* loop iterates at most maximum cycle times, and the *For* loop iterates N times in each iteration as follows:

$$TC(GAP) = O(Max_cycle*N) + O(Max_cycle*L)$$

As a greedy selection mechanism selects the three best solutions, L is equal to N . Then,

$$TC(GAP) = O(2*(Max_cycle*N))$$

- (3) *Objective function computational phase (OFCP):* The objective function is computed for each iteration's d control variable as follows:

$$TC(OFCP) = O(d)$$

- (4) *Solution repair phase (SRP):* The solution obtained for d control variables is involved as follows:

$$TC(OFCP) = O(d)$$

Therefore, the overall TC for finding the optimal solution for EESD is as follows:

5. Simulation and results

5.1 Description of the test system

The GWO is programmed in MATLAB 8.1 environment and simulated on the Intel Core i5 processor personal computer. This study considers two test systems (TS), and the first system embodies 10 generating units encompassing quadratic and non-convex cost characteristics. The load demand is set into 2,000 MW, and network loss is let into a goal (Mandal and Chakraborty, 2018). In the second TS, the unit records of TS-I are customized, and the load demand was scheduled 24 h with an hour interval (Pandit *et al.*, 2012). The numerical results have been demonstrated in two scenarios: static dispatch, whereas the second is dynamic dispatch.

5.2 Parameters setting

The control parameters of the GWO algorithm have been adopted as that of Mirjalili (2014) as follows:

No. of GWO colony size:	30 (population)
Max. iteration:	500
Dimension:	10
Best_score:	Alpha_score
Best_position:	Destination_position

5.3 Cost-effective perspective

The profitable operation speculates the objective of sharing the demand of a power system among the various generation units in such a way as to minimize the FC of the TPS, satisfying the multiple constraints. The efficacy of GWO for solving the CED has been explored by exempting and containing the valve point loading effect on FC characteristics (Kadali *et al.*, 2020). Figure 2 shows the best FC and worst ER associated with the optimal generation to meet the scheduled load demand. It is made aware that the best FC is \$111,140.8317/h and \$111,355.0265/h for with and without valve point effect subjected to static load demand. The corresponding generation schedule is found out about 2,078.9714 and 2,079.0130 MW. Further, it is perceived that the total-generation schedule includes transmission loss, is evolved as 41,062.3270 and 41,065.6206 MW for with and without valve point-effect subjected to static load demand. The corresponding minimum FC is \$2,444,072.4952 and \$2,462,593.6473, 290,749.1277, respectively.

5.4 Environmentally sustainable aspect

With the concern over environmental protection, an attempt is pursued to optimize the generation schedule of thermal units in such a way as to minimize ER, satisfying the operational and practical constraints. The control parameter of the GWO algorithm is suitably turned in terms of a system parameter and has been simulated for minimum pollutant ER for a static load and a time-varying load profile. The static dispatch is optimized 2,066.0756 and 2,066.7114 MW of total generation for without and with valve point effect on the cost curve. At the same time, dynamic dispatch is optimized 41,079.6177

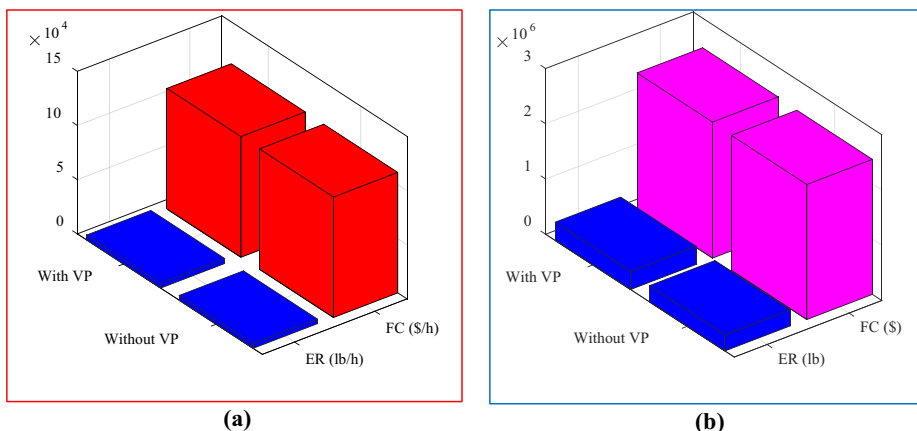


Figure 2.
Best fuel cost

Notes: (a) Static dispatch; (b) dynamic dispatch

and 41,085.0294 MW of total generation. Figure 3 shows the tolerable ER 3,792.9450 and 3,879.5721 lb/h for static dispatch without and with a valve point effect. In comparison, the unobjectionable ER is 269,781.9103 and 290,737.8059 lb during dynamic dispatch without and with valve point effect. In this context, it is fathomed that the optimal generation schedule corresponding to the minimum ER fully satisfies its inequality and power balance constraints adequately while supplying both static and dynamic loads.

5.5 Economic environmentally sustainable dispatch

The total FC's and ER's that GWO has obtained in CED and ED were compared pictorially in Figures 4 and 5 to provide conscience about the operation TPS. It is inferred that the FC is minimal in the case of economic aspects than ER. In the same manner, ER is minimal in the

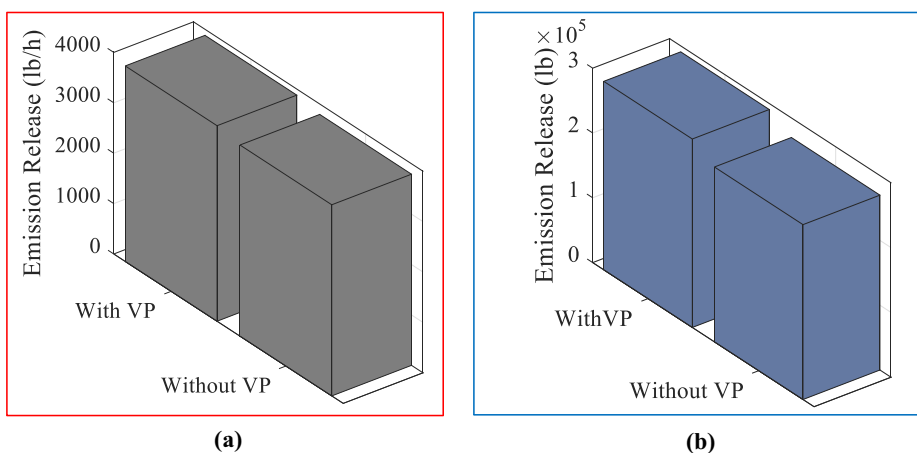


Figure 3.
Minimum emission release

Notes: (a) Static load; (b) dynamic load

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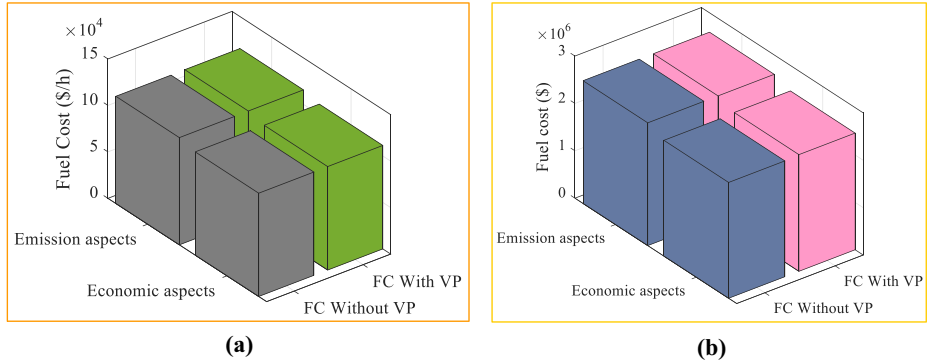


Figure 4.
Comparison of best
fuel cost

Notes: (a) Static dispatch; (b) dynamic dispatch

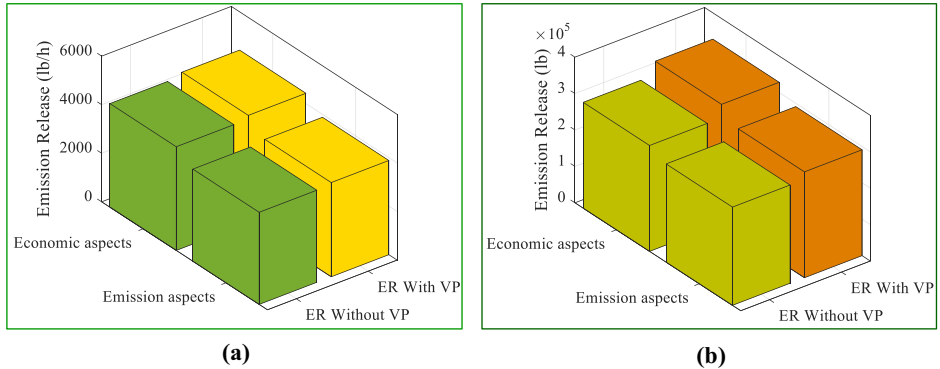


Figure 5.
Comparison of
minimum emission
release

Notes: (a) Static demand; (b) dynamic demand

case of emission aspects than FC. It is proved that economic and emission objectives are conflicting; therefore, both objectives have been optimized simultaneously with static and dynamic load demand.

5.5.1 Static generation schedule for a compromised solution. There are many practical approaches to obtain trade-offs among the non-commensurable objective functions in a multiobjective optimization environment. Particularly in this framework, a linear interpolated price penalty factor approach has been adopted to minimize both conflicting objective functions simultaneously. In this context, the corresponding price penalty factor for a particular scheme is 34.7858 and \$35.5284/lb with and without valve point, respectively. These factors are accurately combined the FC and ER and help provided for optimizing simultaneously. As the objective of EESD is to minimize both the FC and ER simultaneously, the generating schedule is optimized not only between its lower and upper generation limit and also to satisfy the equality constraints. In line with the objective, GWO has tuned for the optimum generation schedule without and with valve point loading and is tabulated in [Table 1](#).

Generation (MW)		Demand 2,000 MW	
		Without valve point	With valve point
Units	P1	42.1821	54.7979
	P2	58.5484	70.8587
	P3	77.7854	87.7660
	P4	82.3326	85.3548
	P5	149.4509	160.0000
	P6	175.4625	203.1300
	P7	257.5695	246.2489
	P8	325.4139	253.5066
	P9	373.5234	384.6749
	P10	470.0000	466.0000
Total generation (MW)		2,012.2687	2,012.3378
Losses (MW)		12.2687	12.3378
Fuel cost (\$/h)		1,09,804.1078	1,11,049.8047
Emission release (lb/h)		3,789.1193	3,811.0333
Iterations		500	

Table 1.
Compromised optimal generation schedule for static dispatch

5.5.2 *Dynamic generation schedule for a compromised solution.* In an inconstant loading environment, a typical task is to optimally dispatch the total load demand to the committed generating units to obtain a compromised point between FC and ER. The normalized price penalty factors corresponding to the non-inferior solution are given in [Table 2](#), facilitating a balance among these non-commensurable objective functions. Subsequently, the optimum

Interval (h)	P_D (MW)	h_k (\$/lb)
1	5,448	6.5539
2	5,776	6.9492
3	5,664	7.2653
4	5,624	7.5815
5	5,928	7.7395
6	6,064	7.9744
7	6,088	8.0071
8	5,856	8.0398
9	5,480	9.8736
10	5,464	11.3397
11	5,728	12.4077
12	5,536	13.4823
13	5,400	11.3397
14	5,828	9.8736
15	3,928	8.0398
16	3,840	7.8976
17	3,784	7.7395
18	3,608	7.9744
19	3,584	8.0398
20	3,544	11.3397
21	3,528	9.8736
22	3,552	7.9744
23	3,688	7.4234
24	3,840	7.1072

Table 2.
Interpolated price penalty factor for compromised dynamic dispatch

hourly generation schedule, total generation and line loss that the GWO algorithm has obtained correspond to the compromised FC \$2,517,985.1046 with tolerable ER 301,302.7248 lb over an entire scheduling time was recorded in [Table 3](#) with valve point loading effect. Numerical values show that power balance, minimum and maximum generation limits are satisfied utterly.

5.6 Competency with other methods

To elevate the proposed algorithm's proficiency in finding compromised dispatch, the feasible solution that has been ascertained by GWO and other methods for solving the multiobjective EESD is compared in this section. The compromised FC and ER are presented in [Table 4](#) for static load, and it is found to be \$111,049.8047/h and 3,811.0333 lb/h using the GWO algorithm best-compromised solution compared with the earlier techniques. To examine the tendency and property of the GWO algorithm while compromising FC and ER, a set of 20 non-dominated solutions obtained by GWO, EMOCA ([Zhang et al., 2013](#)), MODE ([Basu, 2011](#)), NSGA-II ([Zhang et al., 2013](#)), PDE ([Basu, 2011](#)), SPEA ([Basu, 2011](#)) and PHOA ([Rizk-Allah et al., 2018](#)) are compared in [Figure 6](#). It reveals that the proposed algorithm's solutions are distributed widely and have the best diversity on the Pareto optimal front compared with other methods.

In the dynamic load case, the compromised FC in \$ and ER in lb has been compared with other methods in [Table 5](#), whereas the non-dominated solutions are depicted in [Figure 7](#). From the comparison, it can be stated that the GWO method has explored a good quality solution than other methods. At the same time, the GWO algorithm compromises the FC of \$3,872.1046 higher than modified adaptive multiobjective differential evolution algorithm (MAMODE) ([Jiang et al., 2013](#)) and ER 6,181.8632 lb more than NEHS ([Li et al., 2019](#)). Alternatively, it can be interpreted that the MAMODE ([Jiang et al., 2013](#)) attempts to reduce the ER further an amount 1,439.2752 lb, and NEHS ([Li et al., 2019](#)) strives to minimize the FC of \$15,212.0943. Consequently, the FC and ER compromised by MAMODE ([Jiang et al., 2013](#)) and NEHS ([Li et al., 2019](#)), respectively, seem higher than the GWO algorithm has ascertained. It can be evident from the figure that the optimal front proposed by NEHS ([Li et al., 2019](#)) looks local optima because the actual non-dominated front compromises FC and ER in a higher ratio than the GWO algorithm.

5.7 Solution quality improvements

The effectiveness of the proposed method is investigated in terms of solution quality against other competitors. For understanding the ability of GWO in determining EESD, the FC saving and ER attenuated while obtaining trade-offs among them are illustrated in [Figure 8](#) for TS-I and [Figure 9](#) for TS-II. It is perceived from the figures that the GWO algorithm has retrenched FC and alleviated ER apropos of all competitor algorithms.

[Figure 9](#) is developed based on the comprehensive data that have been derived from [Table 5](#) for further analysis. It is noticed that the GWO has saved the FC and diminished ER about NSGA-II ([Basu, 2008](#)) and real-coded genetic algorithm (RCGA) ([Basu, 2008](#)). Alternatively, it saves more FC than modified real-coded genetic algorithm (MRGA) ([Zhu et al., 2016](#)) and NEHS ([Li et al., 2019](#)), whereas reduced remarkable quantity ER relating to CRO ([Roy and Bhui, 2015](#)), DE-SQP ([Elaiw et al., 2013](#)), PSO-SQP ([Elaiw et al., 2013](#)), MAMODE ([Jiang et al., 2013](#)) and MNSGA-II ([Zhu et al., 2016](#)). If MRGA ([Zhu et al., 2016](#)) and NEHS ([Li et al., 2019](#)) algorithms try to minimize FC equal to the value obtained by GWO, the corresponding ER might be higher than what GWO has achieved. Likewise, CRO ([Roy and Bhui, 2015](#)), DE-SQP ([Elaiw et al., 2013](#)), PSO-SQP ([Elaiw et al., 2013](#)), MAMODE ([Jiang et al., 2013](#)) and MNSGA-II ([Zhu et al., 2016](#)) have attempted to minimize ER. Further,

Hours	Generation in MW														PL (MW)
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	Total gen.				
1	135.7792	135.6703	110.0000	110.7796	120.9362	115.0000	101.8351	90.8006	80.0000	55.0000	1,055.8010	19.8010			
2	135.0938	135.0000	128.2391	130.6792	130.4433	120.0000	110.7213	109.3140	79.0269	54.5584	1,133.0760	23.0760			
3	155.0901	149.2338	166.9855	157.0000	143.3070	138.0000	122.7213	120.0000	79.1997	55.0000	1,286.5374	28.5374			
4	156.5282	155.0000	152.9439	213.1742	218.8070	160.0000	130.0000	120.0000	80.0000	55.0000	1,441.4533	35.4533			
5	156.6319	177.0000	187.4664	213.6353	242.6563	159.7867	129.0000	119.4624	79.0380	54.6942	1,519.3712	39.3712			
6	174.5050	172.9531	269.0000	271.1968	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	1,675.6549	47.6549			
7	189.7646	216.4255	289.0368	271.2298	242.8707	160.0000	130.0000	120.0000	80.0000	55.0000	1,754.3274	52.3274			
8	218.0933	253.8214	289.1381	284.3793	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	1,833.4321	57.4321			
9	298.0933	328.7992	289.8279	283.9056	242.3778	159.1502	129.6830	119.4164	79.1898	54.1559	1,984.5991	60.5991			
10	369.1711	374.5167	289.0939	284.1538	242.6746	159.0761	129.5416	119.2685	79.3069	54.7703	2,101.5735	29.5735			
11	382.6229	382.2883	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	2,192.9112	46.9112			
12	404.0601	402.6058	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	2,234.6659	14.6659			
13	369.7918	374.2918	324.3333	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	2,156.4169	84.4169			
14	303.5866	308.8302	299.6849	294.2739	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	1,994.3756	70.3756			
15	223.5866	253.7858	279.5200	274.6595	242.7240	159.9245	129.0180	119.8307	79.3303	54.5572	1,816.9366	40.9366			
16	158.6507	175.0000	243.8352	240.6414	234.6765	159.9276	129.3189	119.1188	79.1857	54.6036	1,594.9584	40.9584			
17	156.4298	169.0104	219.5178	200.0000	242.1633	146.7728	129.2281	119.2347	79.8617	54.4461	1,516.6647	36.6647			
18	178.5337	194.5305	259.0000	236.4450	242.5322	159.8160	129.7865	119.1866	79.2882	54.7875	1,653.9062	25.9062			
19	219.2991	254.0586	279.5995	274.1529	242.5564	159.0497	129.2846	119.7247	78.5723	54.5076	1,810.8054	34.8054			
20	320.2991	354.0586	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	2,102.3577	30.3577			
21	308.6139	313.8030	289.8028	284.6369	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	1,984.8566	60.8566			
22	223.6139	228.8030	177.0000	253.1741	242.8682	159.3270	129.2189	119.6252	79.8607	54.0881	1,667.5791	39.5791			
23	141.5321	146.8030	157.5720	203.1741	162.8682	157.6838	129.3551	119.8507	79.9994	54.1796	1,353.0180	21.0180			
24	131.3930	131.4107	155.6847	153.1741	152.9503	129.0000	119.3551	102.0000	79.3471	54.8944	1,209.2094	25.2094			

Table 3. Hourly compromised generation schedule for dynamic dispatch

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corresponding FC's might be higher than what the GWO has obtained. Therefore, it can be stated that the proposed linear interpolated price penalty approach has been incorporated into the GWO for getting the trade-off value between two objectives correctly correlated to the load demand.

5.8 Statistical measure

5.8.1 Descriptive statistics. In recent decades, many evolutionary and swarm intelligence algorithms have been used in the CEED framework, and they outperform each other numerically. In this situation, the use of standard statistical analysis provides help to investigate the performance of a selected algorithm that would confirm whether a proposed method offers a significant enhancement or not. The statistical analysis data is collected from the trade-off curve, which has attained over 20 independent trial solutions. The analysis results in terms of best, average and worst total FC, minimum, mean and maximum total ER were presented in [Table 6](#) for TS-I and [Table 7](#) for TS-II.

Table 4.
Comparison of
compromised FC and
ER of TS-I supplying
static load

Methods	Fuel cost (\$/h)	Emission (lb/h)
PDE (Basu, 2011)	113,510.0000	4,111.4000
EMOCA (Zhang <i>et al.</i> , 2013)	113,445.0000	4,113.9800
MODE (Basu, 2011)	113,484.0000	4,124.9000
NSGAI (Zhang <i>et al.</i> , 2013)	113,539.0000	4,130.2000
SPEA (Basu, 2011)	113,520.0000	4,109.1000
GSA (Güvenç <i>et al.</i> , 2012)	113,490.0000	4,111.4000
CIHSA (Rezaie <i>et al.</i> , 2018)	116,390.2783	3,932.4473
ABCPSO (Manteaw and Odero, 2012)	113,420.0000	4,120.1000
DE (Basu, 2011)	113,480.0000	4,124.9000
PHOA (Rizk-Allah <i>et al.</i> , 2018)	111,960.0000	3,824.6645
GWO	111,049.8047	3,811.0333

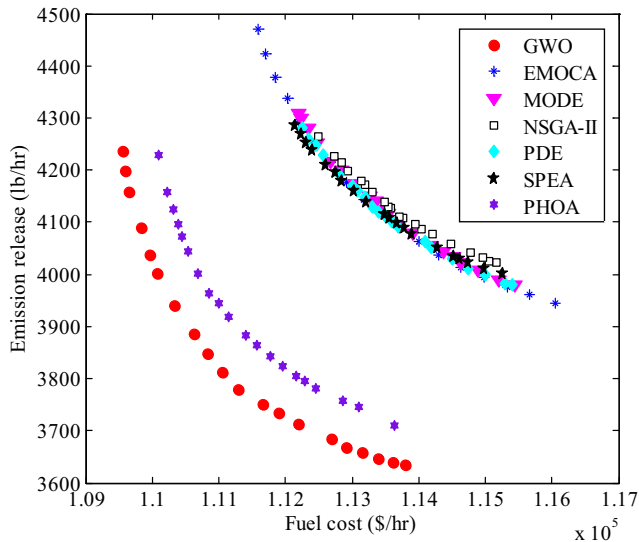


Figure 6.
Comparison of
optimal front for
compromised static
dispatch

Moreover, the associated standard deviation of each attribute is listed in all tables. It is a more informative measure of the statistical dispersion of a data set. It is observed that the GWO has found the best FC and minimum ER with a lesser standard deviation than other methods; this indicates that these attributes are clustered tightly around the average value. Simultaneously, different algorithms have a higher standard deviation than one another; it is not necessarily a bad thing, but it reflects a large variability from their average value. Therefore, it is evident that the GWO algorithm is a good competitor in finding the best solution.

5.8.2 Wilcoxon signed-rank test. It is a non-parametric statistical hypothesis test used to test the null hypothesis of the two samples from the same population against an alternative hypothesis. It is analogous to the paired *t*-test in non-parametric statistical procedures; thus, it is a pair-wise test that purports to find significant deviations between two sample means, i.e. the behavior of two algorithms. The test data are compromised FC and ER obtained by

Methods	Fuel cost (\$)	Emission (lb)
CRO (Roy and Bhui, 2015)	2,517,821.0349	301,941.9173
HCRO (Roy and Bhui, 2015)	2,517,076.3921	299,065.5049
DE-SQP (Elaiw <i>et al.</i> , 2013)	2,468,800.0000	315,640.0000
PSO-SQP (Elaiw <i>et al.</i> , 2013)	2,470,100.0000	315,070.0000
MAMODE (Jiang <i>et al.</i> , 2013)	2,514,113.0000	302,742.0000
IBFA (Pandit <i>et al.</i> , 2012)	2,517,116.7460	299,036.7059
NSGA-II (Basu, 2008)	2,522,600.0000	309,940.0000
RCGA (Basu, 2008)	2,525,100.0000	312,460.0000
MRGA (Zhu <i>et al.</i> , 2016)	2,555,180.8800	299,140.8600
MNSGA-II (Zhu <i>et al.</i> , 2016)	2,517,711.4300	308,674.1500
NEHS (Li <i>et al.</i> , 2019)	2,533,197.1989	295,120.8616
GWO	2,517,985.1046	301,302.7248

Table 5. Comparison of the compromised FC and ER of TS-II supplying dynamic load

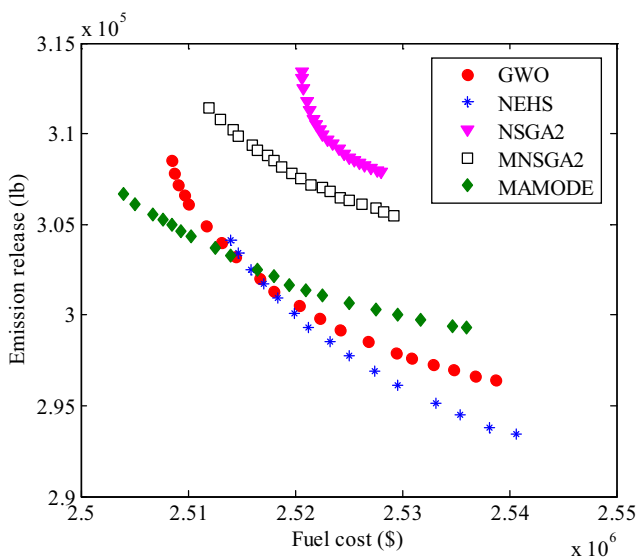


Figure 7. Comparison of trade-off solution for dynamic dispatch

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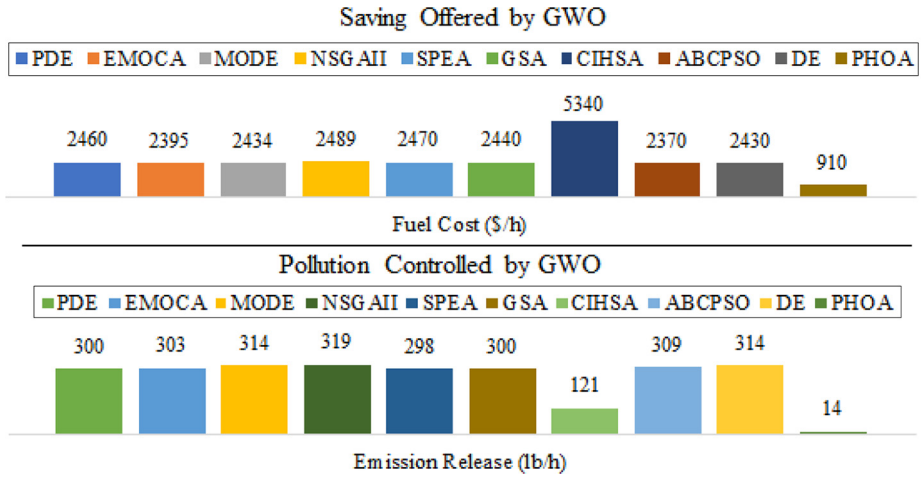


Figure 8.
Competency of GWO
for compromised
static dispatch

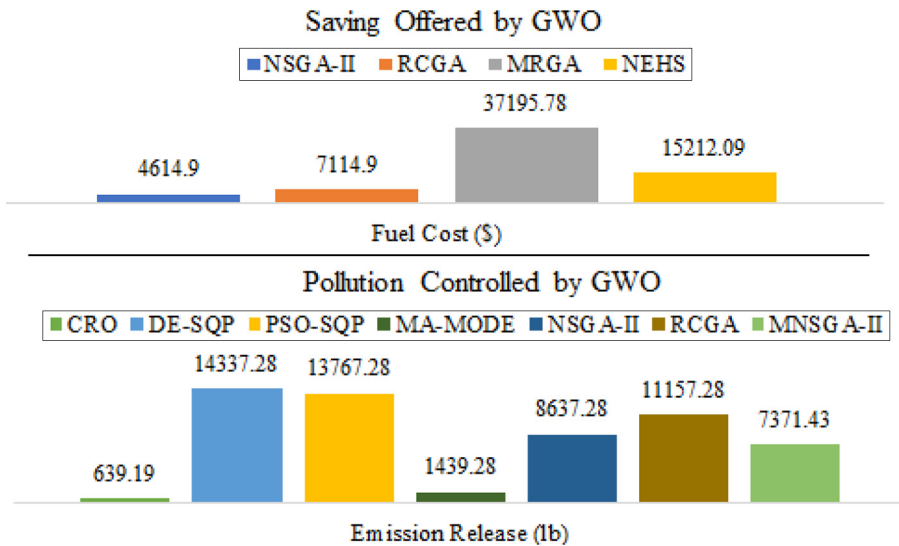


Figure 9.
Competency of GWO
for compromised
dynamic dispatch

GWO and other contestant algorithms for 20 independent trials. The Wilcoxon signed-rank test is performed at the significance level $\alpha = 0.05$, and experimental results were presented in [Table 8](#) for static load demand and [Table 9](#) for dynamic load pattern.

The test statistics comprise the ranks (R^+ and R^-), the standardized test statistic (z), p -values in the asymptotic sig. (two-tailed). It shows that the p -values computed for all the pair-wise comparisons concerning GWO are 0.000 at the significance level $\alpha = 0.05$ or less than 0.05. It is the most substantial evidence to reject the null hypothesis H_0 , which means that the GWO shows a significant improvement to ascertain the best FC and minimum ER over other algorithms, such as EMOCA ([Zhang et al., 2013](#)), MODE ([Basu, 2011](#)), NSGA-II ([Zhang et al., 2013](#)), PDE ([Basu, 2011](#)), SPEA ([Basu, 2011](#)), PHOA ([Rizk-Allah et al., 2018](#)),

Methods	Best	FC (\$/h)			Std. dev.	Minimum	ER (lb/h)			Std. dev
		Average	Worst				Mean	Maximum		
EMOCA (Zhang <i>et al.</i> , 2013)	113,445.00	113,531.76	115,438.83	996.92	4,113.98	4,167.04	4,308.08	97.05		
MODE (Basu, 2011)	113,484.00	113,614.11	115,434.80	975.40	4,124.90	4,129.10	4,308.75	91.93		
NSGA-II (Zhang <i>et al.</i> , 2013)	113,539.00	113,769.32	115,157.74	809.37	4,130.20	4,123.20	4,299.41	84.61		
PDE (Basu, 2011)	113,510.00	113,621.50	115,400.00	945.00	4,111.40	4,117.90	4,280.00	84.30		
SPEA (Basu, 2011)	113,520.00	113,516.00	115,250.00	865.00	4,109.10	4,128.65	4,286.00	88.45		
PHOA (Rizk-Allah <i>et al.</i> , 2018)	111,960.00	111,445.50	113,640.00	840.00	3,824.67	3,927.82	4,017.67	96.50		
GWO	111,049.81	111,411.00	112,497.62	723.91	3,811.03	3,855.43	3,977.54	83.26		

Table 6.
Descriptive statistics
for compromised FC
and ER of TS-I
supplying static load

Grey wolf
optimizer

Table 7.
Descriptive statistics
for compromised FC
and ER of TS-II
supplying dynamic
load

Methods	FC (\$)				ER (lb)			
	Best	Average	Worst	Std. dev.	Minimum	Mean	Maximum	Std. dev
MNSGA-II (Zhu <i>et al.</i> , 2016)	2,517,711.43	2,520,430.00	2,529,200.00	5,744.28	308,674.15	307,930.00	312,400.00	1,862.92
NSGA-II (Basu, 2008)	2,522,600.00	2,523,718.33	2,532,033.33	4,716.67	309,940.00	309,914.50	313,416.67	1,738.33
MAMODE (Jiang <i>et al.</i> , 2013)	2,514,113.00	2,518,007.50	2,526,000.00	5,943.50	302,742.00	302,634.94	306,666.67	1,962.33
NEHS (Li <i>et al.</i> , 2019)	2,533,197.20	2,534,120.00	2,546,000.00	6,401.40	295,120.86	300,830.00	304,000.00	4,439.57
GWO	2,517,985.11	2,520,883.60	2,524,784.99	3,399.94	301,302.73	301,620.72	304,501.53	1,599.40

Pair-wise comparison	Test statistics ^a							
	FC (\$/h)				ER (lb/h)			
	Sum of the rank		Z	Asymp. sig. (two-tailed) p-value	Sum of the rank		Z	Asymp. sig. (two-tailed) p-value
R ⁺	R ⁻	R ⁺			R ⁻			
GWO-EMOCA (Zhang <i>et al.</i> , 2013)	0	20	-3.920 ^b	0.000	0	20 ^a	-3.921 ^b	0.000
GWO-MODE (Basu, 2011)	0	20	-3.920 ^b	0.000	0	20 ^a	-3.921 ^b	0.000
GWO-NSGAI (Zhang <i>et al.</i> , 2013)	0	20	-3.920 ^b	0.000	0	20 ^a	-3.920 ^b	0.000
GWO-PDE (Basu, 2011)	0	20	-3.920 ^b	0.000	0	20 ^a	-3.920 ^b	0.000
GWO-SPEA (Basu, 2011)	0	20	-3.920 ^b	0.000	0	20 ^a	-3.920 ^b	0.000
GWO-PHOA (Rizk-Allah <i>et al.</i> , 2018)	2	18	-2.576 ^b	0.010	2	18	-2.613 ^b	0.009
GWO-GSA (Güvenç <i>et al.</i> , 2012)	1	19	-3.883 ^b	0.000	0	20	-3.920 ^b	0.000
GWO-CIHS (Rezaie <i>et al.</i> , 2018)	0	20	-3.920 ^b	0.000	0	20	-3.921 ^b	0.000
GWO-ABCPSO (Manteaw and Odero, 2012)	0	20	-3.920 ^b	0.000	3	17	-3.621 ^b	0.000
GWO-DE (Basu, 2011)	0	20	-3.920 ^b	0.000	3	17	-3.510 ^b	0.000

Table 8. Wilcoxon signed-rank test statistics for compromised FC and ER of TS-I supplying static load

Notes: ^aWilcoxon signed-ranks test; ^bbased on positive ranks

Pair-wise comparison	Test statistics ^a							
	FC (\$)				ER (lb)			
	Sum of the rank		Z	Asymp. sig. (two-tailed) p-value	Sum of the rank		Z	Asymp. sig. (two-tailed) p-value
R ⁺	R ⁻	R ⁺			R ⁻			
GWO-NEHS (Li <i>et al.</i> , 2019)	5	15	-2.875 ^b	0.004	0	20	-3.920 ^b	0.000
GWO-NSGA-II (Basu, 2008)	2	18	-2.539 ^b	0.011	1	19	-3.845 ^b	0.000
GWO-MNSGA-II (Zhu <i>et al.</i> , 2016)	16	4	-2.091 ^c	0.037	0	20	-3.921 ^b	0.000
GWO-MAMODE (Jiang <i>et al.</i> , 2013)	18	2	-2.576 ^c	0.010	4	16	-3.360 ^b	0.001
GWO-CRO (Roy and Bhui, 2015)	5	15	-3.173 ^b	0.002	5	15	-3.323 ^b	0.001
GWO-HCRO (Roy and Bhui, 2015)	5	15	-3.099 ^b	0.002	4	16	-3.397 ^b	0.001
GWO-DE-SQP (Elaiw <i>et al.</i> , 2013)	20	0	-3.920 ^c	0.000	0	20	-3.921 ^b	0.000
GWO-PSO-SQP (Elaiw <i>et al.</i> , 2013)	20	0	-3.920 ^c	0.000	1	19	-3.883 ^b	0.000
GWO-IBFA (Pandit <i>et al.</i> , 2012)	5	15	-3.099 ^b	0.002	17	3	-2.501 ^c	0.012
GWO-RCGA (Basu, 2008)	1	19	-3.845 ^b	0.000	2	18	-3.808 ^b	0.000
GWO-MRGA (Zhu <i>et al.</i> , 2016)	0	20	-3.920 ^b	0.000	17	3	-2.355 ^c	0.019

Table 9. Wilcoxon signed-rank test statistics for compromised FC and ER of TS-II supplying dynamic load

Notes: ^aWilcoxon signed-ranks test; ^bbased on positive ranks; ^cbased on negative ranks

GSA (Güvenç *et al.*, 2012), CIHS (Rezaie *et al.*, 2018), ABCPSO (Manteaw and Odero, 2012) and DE (Basu, 2011) for static load demand. At the same time, NEHS (Li *et al.*, 2019), NSGA-II (Basu, 2008), MNSGA-II (Zhu *et al.*, 2016), MAMODE (Jiang *et al.*, 2013), CRO (Roy and Bhui, 2015), HCRO (Roy and Bhui, 2015), DE-SQP (Elaiw *et al.*, 2013), PSO-SQP (Elaiw *et al.*, 2013), IBFA (Pandit *et al.*, 2012), RCGA (Basu, 2008) and MRGA (Zhu *et al.*, 2016) for dynamic load demand.

6. Conclusion

It is summarized that EESD is framed as a non-linear optimization problem. The objective functions are the FC and ER of thermal units that included the valve point loading effect in

the cost function and modeled both the static and dynamic load environments. The problem formulation has considered the power balance equation and lower and upper generation limits; moreover, POZ and up/down RRLs for realistic operation. It is found that the FC and ER are conflicting. Therefore, an efficient analytical geometry procedure is incorporated into the price penalty factor, and a linear interpolated price penalty approach is developed, which has blended the non-commensurable objectives perfectly in correlation with load demand. The power balance equation has been handled effectively using a solution repair strategy, whereas POZ and RRL are appropriately handled during the iteration. A well-known two TSs are considered in this study.

A proficient metaheuristic GWO algorithm is exercised as an optimization tool. Though constraints handling mechanisms fluctuate computational time, the GWO discovers global optimum solutions because of self-organization and natural leadership hierarchy. The performance of the GWO algorithm in minimizing stated objectives are compared as follows with well-known optimization techniques that have already proven their ability in solving EESD:

- It has offered better saving of FC.
- In terms of environmental aspects, it has been remarkably reduced pollutant emission.
- To a great extent, GWO has obtained an effective trade-off between FC and ER. Thus, it affords good FC saving with the massive reduction in ER while EESD.

Further, the numerical results provide the following pertinent acquaintance on scheming TPS. The valve point loading increases the power generation; thus, the FC increases than without valve point loading in all operations, which seems to be practical value. Finally, it is concluded that the GWO provides the best solution for economic environmentally sustainable generation schedule and is very useful to the system planners. This way, it alters their working methodologies to produce electrical vitality at affordable prices with cleanliness.

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